More on Supervised Learning

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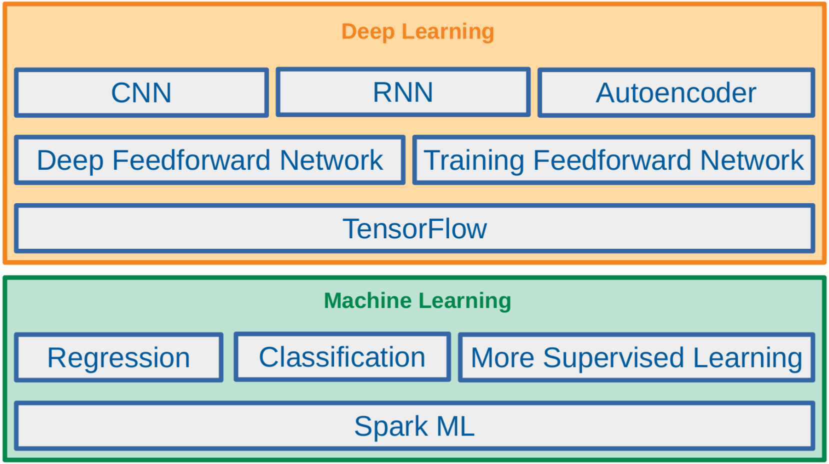
21/11/2018

The Course Web Page

`https://id2223kth.github.io`

Where Are We?


Deep Learning
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More Supervised Learning
Spark ML

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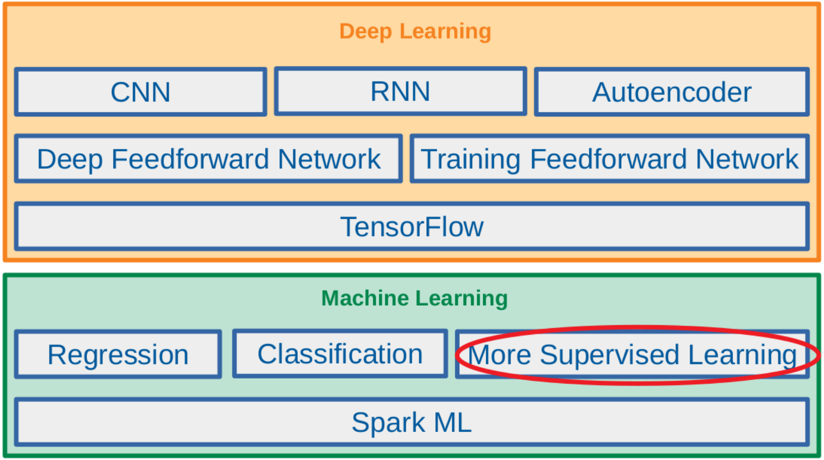
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Let's Start with an Example

Buying Computer Example (1/3)

- Given the dataset of m people.

id	age	income	student	credit rating	buys computer
1	youth	high	no	fair	no
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- Predict if a new person buys a computer?
- Given an instance $\mathbf{x}^{(i)}$, e.g., $x_1^{(i)} = \mathtt{senior}$, $x_2^{(i)} = \mathtt{medium}$, $x_3^{(i)} = \mathtt{no}$, and $x_4^{(i)} = \mathtt{fair}$, then $y^{(i)} = ?$

Buying Computer Example (2/3)

id	age	income	student	credit rating	buys computer
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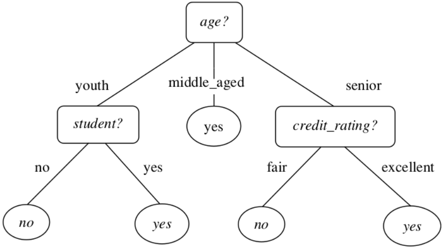

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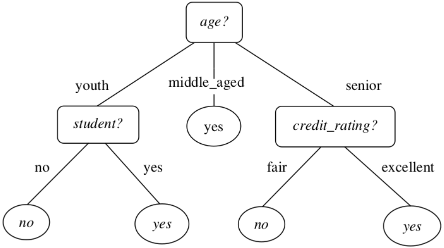

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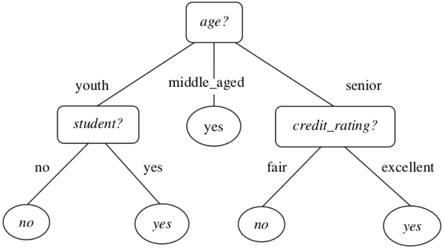

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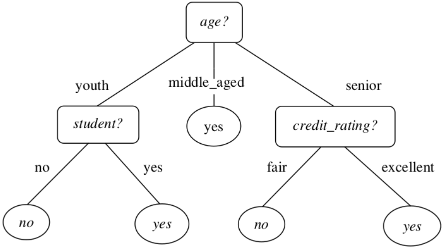

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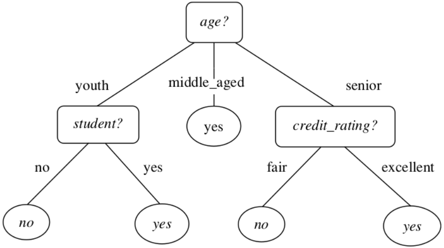
- E.g., input $\mathbf{x}^{(i)}$ with $x_1^{(i)} = \texttt{senior}$, $x_2^{(i)} = \texttt{medium}$, $x_3^{(i)} = \texttt{no}$, and $x_4^{(i)} = \texttt{fair}$.



Decision Tree

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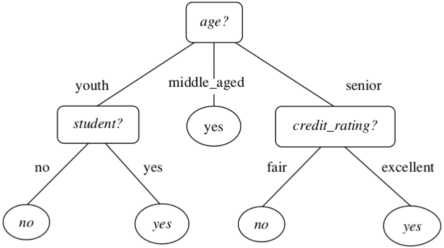

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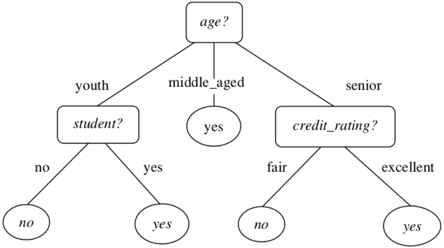

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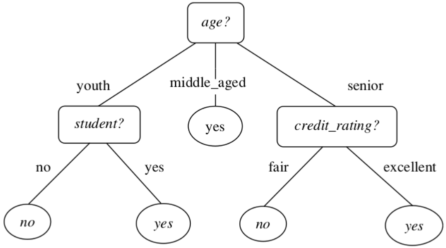

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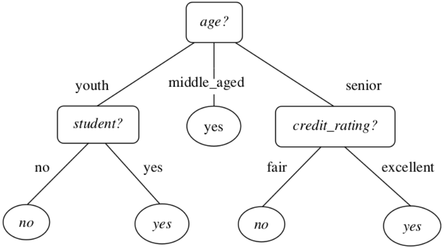

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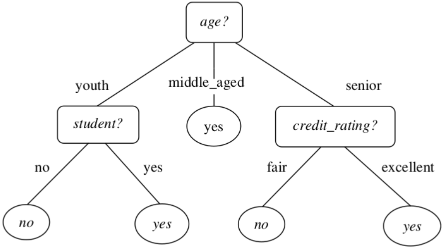

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- 4. The algorithm repeats the same process recursively to form a decision tree.

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- In conditions 2 and 3:
  - Convert node N into a leaf.
  - Label it either with the most common class in D.
  - Or, the class distribution of the node tuples may be stored.

Training Algorithm - Partitioning Instances (1/3)

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- Three possibilities to partition instances in `D` based on the feature `A`.
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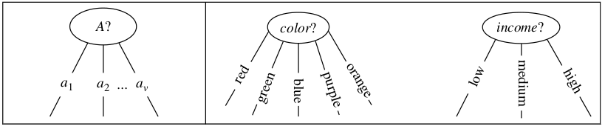

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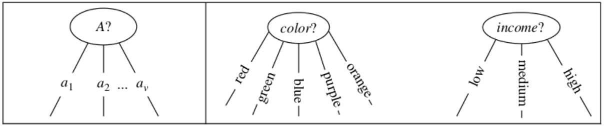

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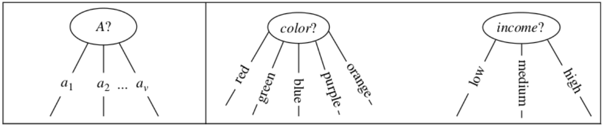
  - Partition D_j is the subset of tuples in D having value a_j of A.



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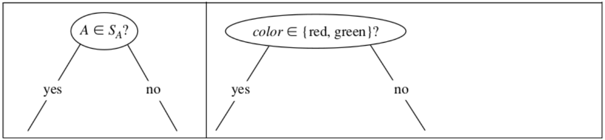

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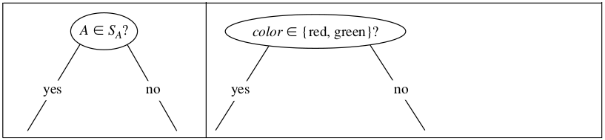

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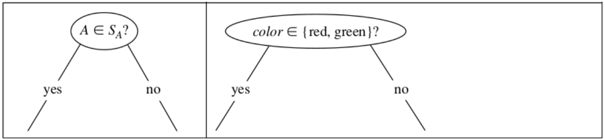

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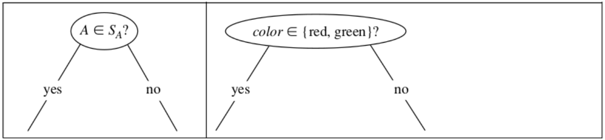

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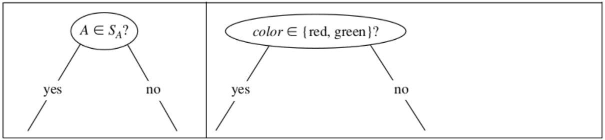

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  - Two branches are labeled according to the previous outcomes.

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- Feature selection measure: how to split instances at a node `N`.
- Pure partiton: if all instances in a partition belong to the same class.
- The best splitting criterion is the one that most closely results in a pure scenario.

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- Two popular feature selection measures are:
  - Information gain (ID3 and C4.5)
  - Gini index (CART)

Information Gain (Entropy)

ID3 (1/8)

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- The feature with the highest information gain is chosen as the splitting feature for node `N`.
- The information gain is based on the decrease in entropy after a dataset is split on a feature.

ID3 (2/8)

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$$\text{entropy}(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

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- D's entropy is zero when it contains instances of only one class (pure partition).

ID3 (3/8)

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2	youth	high	no	excellent	no
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8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

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$$\mathtt{entropy}(\mathtt{D}) = -\sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{p_i} \log_2(\mathtt{p_i})$$

$$\text{label} = \mathtt{buys\_computer} \Rightarrow \mathtt{m} = 2$$

$$\mathtt{entropy}(\mathtt{D}) = -\frac{9}{14}\log_2(\frac{9}{14}) - \frac{5}{14}\log_2(\frac{5}{14}) = 0.94$$

ID3 (4/8)

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- The expected information required to classify an instance from $\mathtt{D}$ based on the partitioning by $\mathtt{A}$ is:

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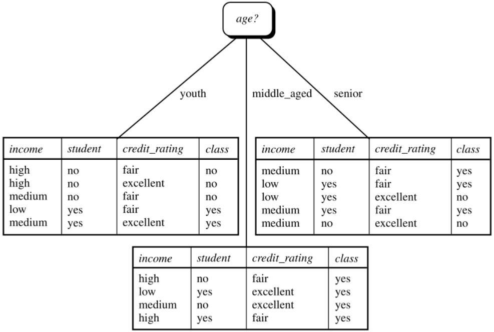
- $\frac{|\mathtt{D}_\mathtt{j}|}{\mathtt{D}}$ is the weight of the $\mathtt{j}$th partition.
- The smaller the expected information required, the greater the purity of the partitions.

ID3 (5/8)



youth:

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high	no	fair	no
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medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

middle_aged:

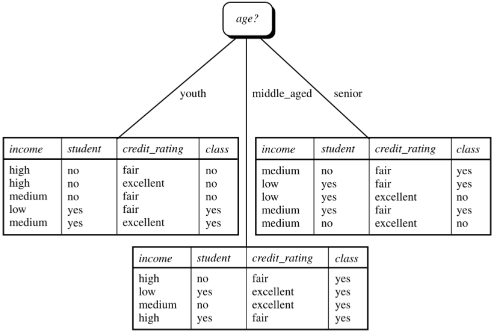
income	*student*	*credit_rating*	*class*
high	no	fair	yes
low	yes	excellent	yes
medium	no	excellent	yes
high	yes	fair	yes

senior:

income	*student*	*credit_rating*	*class*
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

$$\mathtt{entropy}(\mathtt{A}, \mathtt{D}) = \sum_{\mathtt{j}=1}^{\mathtt{v}} \frac{|\mathtt{D_j}|}{|\mathtt{D}|} \mathtt{entropy}(\mathtt{D_j})$$

ID3 (5/8)



youth:

income	*student*	*credit_rating*	*class*
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

middle_aged:

income	*student*	*credit_rating*	*class*
high	no	fair	yes
low	yes	excellent	yes
medium	no	excellent	yes
high	yes	fair	yes

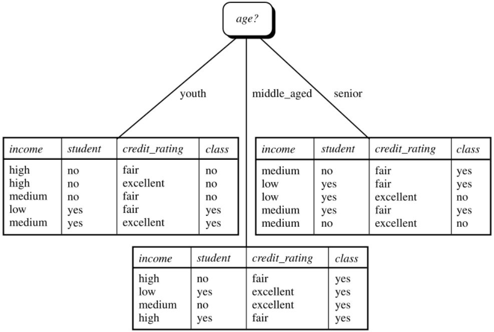
senior:

income	*student*	*credit_rating*	*class*
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

$$\mathtt{entropy}(\mathtt{A},\mathtt{D}) = \sum_{j=1}^{v} \frac{|\mathtt{D}_j|}{|\mathtt{D}|}\mathtt{entropy}(\mathtt{D}_j)$$

$$\mathtt{entropy}(\mathtt{age},\mathtt{D}) = \frac{5}{14}\mathtt{entropy}(\mathtt{D}_{\mathtt{youth}}) + \frac{4}{14}\mathtt{entropy}(\mathtt{D}_{\mathtt{middle\_aged}}) + \frac{5}{14}\mathtt{entropy}(\mathtt{D}_{\mathtt{senior}})$$

ID3 (5/8)



income	*student*	*credit_rating*	*class*
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
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medium	yes	excellent	yes

income	*student*	*credit_rating*	*class*
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$$\texttt{entropy}(\texttt{age},\texttt{D}) = \frac{5}{14}(-\frac{2}{5}\log_2(\frac{2}{5}) - \frac{3}{5}\log_2(\frac{3}{5})) + \frac{4}{14}(-\frac{4}{4}\log_2(\frac{4}{4})) + \frac{5}{14}(-\frac{3}{5}\log_2(\frac{3}{5}) - \frac{2}{5}\log_2(\frac{2}{5})) = 0.694$$

ID3 (6/8)

- The information gain $\texttt{Gain}(\texttt{A}, \texttt{D})$ is defined as:

$$\texttt{Gain}(\texttt{A}, \texttt{D}) = \texttt{entropy}(\texttt{D}) - \texttt{entropy}(\texttt{A}, \texttt{D})$$

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- It shows how much would be gained by branching on $\texttt{A}$.
- The feature $\texttt{A}$ with the highest $\texttt{Gain}(\texttt{A}, \texttt{D})$ is chosen as the splitting feature at node $\texttt{N}$.

ID3 (7/8)

- Now, we can compute the information gain $\texttt{Gain(A)}$ for the feature $\texttt{A = age}$.

$$\texttt{Gain}(\texttt{age}, \texttt{D}) = \texttt{entropy}(\texttt{D}) - \texttt{entropy}(\texttt{age}, \texttt{D}) = 0.940 - 0.694 = 0.246$$

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- Similarly we have:
  - $\texttt{Gain}(\texttt{income},\texttt{D}) = 0.029$
  - $\texttt{Gain}(\texttt{student},\texttt{D}) = 0.151$
  - $\texttt{Gain}(\texttt{credit\_rating},\texttt{D}) = 0.048$

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  - $\texttt{Gain}(\texttt{student}, \texttt{D}) = 0.151$
  - $\texttt{Gain}(\texttt{credit\_rating}, \texttt{D}) = 0.048$

- The age has the highest information gain among the attributes, it is selected as the splitting feature.

ID3 (8/8)

- The bias problem: information gain prefers to select features having a large number of values.

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RID	age	income	student	credit_rating	Class: buys_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

- For example, a split on $\mathtt{RID}$ would result in a large number of partitions.
  - Each partition is pure.
  - Info product $\mathtt{entropy}(\mathtt{RID}, \mathtt{D}) = 0$, thus, the information gained by partitioning on this feature is maximal.

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  - Each partition is pure.
  - Info product $\texttt{entropy}(\texttt{RID}, \texttt{D}) = 0$, thus, the information gained by partitioning on this feature is maximal.
- Clearly, such a partitioning is useless for classification.

C4.5 (1/2)

- C4.5 is a successor of ID3 that overcomes its bias problem.
- It normalizes the information gain using a split information value:

$$\mathtt{SplitInfo}(\mathtt{A},\mathtt{D}) = -\sum_{\mathtt{j}=1}^{\mathtt{v}} \frac{|\mathtt{D_j}|}{|\mathtt{D}|}\log_2\left(\frac{|\mathtt{D_j}|}{|\mathtt{D}|}\right)$$

$$\mathtt{GainRatio}(\mathtt{A},\mathtt{D}) = \frac{\mathtt{Gain}(\mathtt{A},\mathtt{D})}{\mathtt{SplitInfo}(\mathtt{A},\mathtt{D})}$$

C4.5 (2/2)

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6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
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$$\mathtt{SplitInfo}(\mathtt{A},\mathtt{D}) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \log_2\left(\frac{|D_j|}{|D|}\right)$$

$$\mathtt{SplitInfo}(\mathtt{income},\mathtt{D}) = -\frac{4}{14}\log_2\left(\frac{4}{14}\right) - \frac{6}{14}\log_2\left(\frac{6}{14}\right) - \frac{4}{14}\log_2\left(\frac{4}{14}\right) = 1.557$$

- $\mathtt{Gain}(\mathtt{income},\mathtt{D}) = 0.029$, therefore $\mathtt{GainRatio}(\mathtt{income},\mathtt{D}) = \frac{0.029}{1.557} = 0.019$.

Gini Impurity

CART (1/8)

- CART (Classification And Regression Tree) considers a binary split for each feature.

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- $\mathtt{p}_{\mathtt{i}}$ is the probability that an instance in $\mathtt{D}$ belongs to class $\mathtt{i}$, with $\mathtt{m}$ distinct classes.
- It will be zero if all partitions are pure. Why?
- We need to determine the splitting criterion: splitting feature + splitting subset.

CART (2/8)

- Assume $\mathtt{A}$ is a discrete-valued feature with $\mathtt{v}$ distinct values, $\{\mathtt{a}_1, \mathtt{a}_2, \cdots, \mathtt{a}_\mathtt{v}\}$, occurring in $\mathtt{D}$.

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- $\mathtt{S}_\mathtt{A}$ will be all possible subsets of $\mathtt{A}$.
  - E.g., $\mathtt{A} = \mathtt{income} = \{\mathtt{low}, \mathtt{medium}, \mathtt{high}\}$
  - $\mathtt{S}_\mathtt{A} = \{\{\mathtt{low}, \mathtt{medium}, \mathtt{high}\}, \{\mathtt{low}, \mathtt{medium}\}, \{\mathtt{medium}, \mathtt{high}\}, \{\mathtt{low}, \mathtt{high}\}, \{\mathtt{low}\}, \{\mathtt{medium}\}, \{\mathtt{high}\}, \{\}\}$

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  - The test is of the form $\mathtt{D}_1 \in \mathtt{s}_\mathtt{A}$?, where $\mathtt{s}_\mathtt{A}$ is a subset of $\mathtt{S}_\mathtt{A}$, e.g., $\mathtt{s}_\mathtt{A} = \{\mathtt{low}, \mathtt{high}\}$.

CART (3/8)

RID	age	income	student	credit_rating	Class: buys_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
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$$\mathtt{Gini}(\mathtt{D}) = 1 - \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{p}_{\mathtt{i}}^2$$

$$\text{label} = \mathtt{buys\_computer} \Rightarrow \mathtt{m} = 2$$

$$\mathtt{Gini}(\mathtt{D}) = 1 - (\frac{9}{14})^2 - (\frac{5}{14})^2 = 0.459$$

CART (4/8)

- If a binary split on A partitions D into D_1 and D_2, the Gini index of D given that partitioning is:

$$\mathtt{Gini}(A, D) = \frac{|D_1|}{D}\mathtt{Gini}(D_1) + \frac{|D_2|}{D}\mathtt{Gini}(D_2)$$

CART (4/8)

- If a binary split on $\mathtt{A}$ partitions $\mathtt{D}$ into $\mathtt{D}_1$ and $\mathtt{D}_2$, the Gini index of $\mathtt{D}$ given that partitioning is:

$$\mathtt{Gini}(\mathtt{A}, \mathtt{D}) = \frac{|\mathtt{D}_1|}{\mathtt{D}} \mathtt{Gini}(\mathtt{D}_1) + \frac{|\mathtt{D}_2|}{\mathtt{D}} \mathtt{Gini}(\mathtt{D}_2)$$

- The subset that gives the minimum Gini index is selected as its splitting subset.

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- Assume, we choose the splitting subset $\mathtt{s_A} = \{\mathtt{low}, \mathtt{medium}\}$.

CART (5/8)

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- Assume, we choose the splitting subset $\mathtt{s_A} = \{\mathtt{low}, \mathtt{medium}\}$.
- Consider partition $\mathtt{D_1}$ satisfies the condition $\mathtt{D_1} \in \mathtt{s_A}$, and $\mathtt{D_2}$ does not.

$$\mathtt{Gini}_{\mathtt{income}\in\{\mathtt{low},\mathtt{medium}\}}(\mathtt{A}, \mathtt{D}) = \frac{10}{14}\mathtt{Gini}(\mathtt{D_1}) + \frac{4}{14}\mathtt{Gini}(\mathtt{D_2})$$

$$= \frac{10}{14}\mathtt{Gini}(1 - (\frac{7}{10})^2 - (\frac{3}{10})^2) + \frac{4}{14}(1 - (\frac{2}{4})^2 - (\frac{2}{4})^2) = 0.443$$

CART (6/8)

- Similarly, we calculate the Gini index values for splits on the remaining subsets.

$$\mathtt{Gini}_{\mathtt{income}\in\{\mathtt{low},\mathtt{medium}\}}(\mathtt{A},\mathtt{D}) = \mathtt{Gini}_{\mathtt{income}\in\{\mathtt{high}\}}(\mathtt{A},\mathtt{D}) = 0.443$$
$$\mathtt{Gini}_{\mathtt{income}\in\{\mathtt{low},\mathtt{high}\}}(\mathtt{A},\mathtt{D}) = \mathtt{Gini}_{\mathtt{income}\in\{\mathtt{medium}\}}(\mathtt{A},\mathtt{D}) = 0.458$$
$$\mathtt{Gini}_{\mathtt{income}\in\{\mathtt{medium},\mathtt{high}\}}(\mathtt{A},\mathtt{D}) = \mathtt{Gini}_{\mathtt{income}\in\{\mathtt{low}\}}(\mathtt{A},\mathtt{D}) = 0.450$$

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$$\mathtt{Gini}_{\mathtt{income}\in\{\mathtt{medium},\mathtt{high}\}}(\mathtt{A},\mathtt{D}) = \mathtt{Gini}_{\mathtt{income}\in\{\mathtt{low}\}}(\mathtt{A},\mathtt{D}) = 0.450$$

- The best binary split for attribute $\mathtt{A} = \mathtt{income}$ is on $\mathtt{s}_\mathtt{A} = \{\mathtt{low}, \mathtt{medium}\}$ because it minimizes the Gini index.

CART (7/8)

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- The reduction in impurity that would be incurred by a binary split on feature $\mathtt{A}$ is:

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$$\Delta\mathtt{Gini}(\mathtt{A}) = \mathtt{Gini}(\mathtt{D}) - \mathtt{Gini}(\mathtt{A}, \mathtt{D})$$

- The feature that maximizes the reduction in impurity (has the minimum Gini index) is selected as the splitting feature.

CART (8/8)

- Now, we can compute the information gain $\texttt{Gain(A)}$ for different features.
  - $\Delta\texttt{Gini(income)} = 0.459 - 0.443 = 0.016$
  - $\Delta\texttt{Gini(age)} = 0.459 - 0.357 = 0.102$
  - $\Delta\texttt{Gini(student)} = 0.459 - 0.367 = 0.092$
  - $\Delta\texttt{Gini(credit\_rating)} = 0.459 - 0.429 = 0.03$

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  - $\Delta\texttt{Gini(student)} = 0.459 - 0.367 = 0.092$
  - $\Delta\texttt{Gini(credit\_rating)} = 0.459 - 0.429 = 0.03$
- The feature $\texttt{A} = \texttt{age}$ and splitting subset $\texttt{s}_\texttt{A} = \{\texttt{youth}, \texttt{senior}\}$ gives the minimum Gini index overall.

Decision Tree in Spark (1/4)

- Two classes in `spark.ml`.
- Regression: `DecisionTreeRegressor`

```
val dt_regressor = new DecisionTreeRegressor().setLabelCol("label").setFeaturesCol("features")
val model = dt_regressor.fit(trainingData)
val predictions = model.transform(testData)
predictions.select("prediction", "rawPrediction", "probability", "label", "features").show(5)
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- `probabilityCol` is a vector of length of number of classes equal to `rawPrediction` normalized to a multinomial distribution.

Decision Tree in Spark (3/4)

- Tunable parameters

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- `maxBins`: number of bins used when discretizing continuous features.

Decision Tree in Spark (3/4)

- Tunable parameters
- `maxBins`: number of bins used when discretizing continuous features.
- `impurity`: impurity measure used to choose between candidate splits, e.g., `entropy` and `gini`.

```
val maxBins = ...
val dt_classifier = new DecisionTreeClassifier().setMaxBins(maxBins).setImpurity("gini")
```

Decision Tree in Spark (4/4)

- Stopping criteria that determines when the tree stops building.

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Decision Tree in Spark (4/4)

- Stopping criteria that determines when the tree stops building.
- `maxDepth`: maximum depth of a tree.
- `minInstancesPerNode`: for a node to be split further, each of its children must receive at least this number of training instances.
- `minInfoGain`: for a node to be split further, the split must improve at least this much (in terms of information gain).

```
val maxDepth = ...
val minInstancesPerNode = ...
val minInfoGain = ...
val dt_classifier = new DecisionTreeClassifier()
                    .setMaxDepth(maxDepth)
                    .setMinInstancesPerNode(minInstancesPerNode)
                    .setMinInfoGain(minInfoGain)
```

Ensemble Methods

Wisdom of the Crowd

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- In many cases, this aggregated answer is better than an expert's answer.
- This is called the wisdom of the crowd.
- Similarly, the aggregated estimations of a group of estimators (e.g., classifiers or regressors), often gets better estimations than with the best individual estimator.
- A group of estimators is an ensemble, and this technique is called Ensemble Learning.

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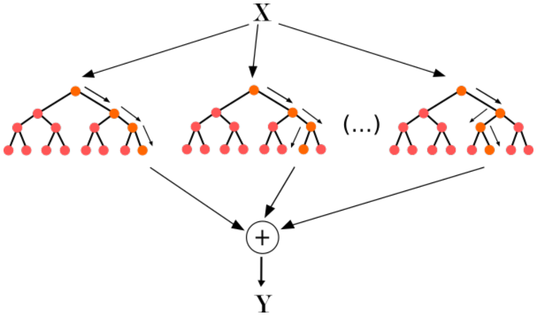
  - E.g., random forest
- Boosting
  - Train estimators sequentially, each trying to correct its predecessor.
  - E.g., adaboost and gradient boosting

Random Forest

- Random forest builds multiple decision trees that are most of the time trained with the bagging method.
- It, then, merges the trees together to get a more accurate and stable prediction.



Random Forest in Spark (1/2)

- Two classes in `spark.ml`.
- Regression: `RandomForestRegressor`

```
val rf_regressor = new RandomForestRegressor().setLabelCol("label")
                                              .setFeaturesCol("features").setNumTrees(10)
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predictions.select("prediction", "label", "features").show(5)
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Random Forest in Spark (2/2)

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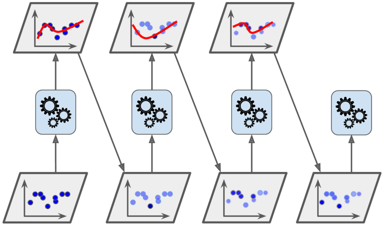
- `subsamplingRate`: specifies the size of the dataset used for training each tree in the forest, as a fraction of the size of the original dataset.
  - Default is 1.0 and decreasing it can speed up training.
- `featureSubsetStrategy`: number of features to use as candidates for splitting at each tree node, as a fraction of the total number of features.
  - Possible values: `auto`, `all`, `onethird`, `sqrt`, `log2`, `n`

AdaBoost (1/3)

- AdaBoost: train a new estimator by paying more attention to the training instances that the predecessor underfitted.



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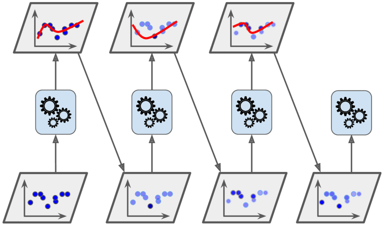
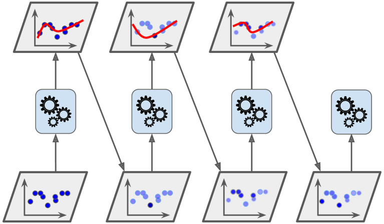


AdaBoost (1/3)

- AdaBoost: train a new estimator by paying more attention to the training instances that the predecessor underfitted.
- Each estimator is trained on a random subset of the total training set.
- AdaBoost assigns a weight to each training instance, which determines the probability that each instance should appear in the training set.



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AdaBoost (2/3)

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$$\mathtt{r}_\mathtt{j} = \frac{\sum_{\mathtt{i}=1, \hat{\mathtt{y}}_\mathtt{j}^{(\mathtt{i})} \neq \mathtt{y}_\mathtt{j}^{(\mathtt{i})}}^{\mathtt{m}} \mathtt{h}^{(\mathtt{i})}}{\sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{h}^{(\mathtt{i})}}$$

- The $\mathtt{j}$th estimator's weight $\alpha_\mathtt{j}$ is then computed as follows:

$$\alpha_\mathtt{j} = \eta \frac{1 - \mathtt{r}_\mathtt{j}}{\mathtt{r}_\mathtt{j}}$$

AdaBoost (3/3)

- Next the instance weights are updated:

$$\mathtt{h}^{(\mathtt{i})} = \begin{cases} \mathtt{h}^{(\mathtt{i})} & \text{if} \quad \hat{\mathtt{y}}_{\mathtt{j}}^{(\mathtt{i})} = \mathtt{y}_{\mathtt{j}}^{(\mathtt{i})} \\ \mathtt{h}^{(\mathtt{i})}\mathtt{e}^{\alpha_{\mathtt{j}}} & \text{if} \quad \hat{\mathtt{y}}_{\mathtt{j}}^{(\mathtt{i})} \neq \mathtt{y}_{\mathtt{j}}^{(\mathtt{i})} \end{cases}$$

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- Then, a new estimator is trained using the updated weights, and the whole process is repeated.

- To make predictions, AdaBoost computes the predictions of all the estimators and weighs them using the estimator weights α_j.

Gradient Boosting (1/3)

- Just like AdaBoost, Gradient Boosting works by sequentially adding estimators to an ensemble, each one correcting its predecessor.
- However, instead of tweaking the instance weights at every iteration, this method tries to fit the new estimator to the residual errors made by the previous estimator.

Gradient Boosting (2/3)

- Let's go through a regression example using Gradient Boosted Regression Trees.
- Fit the first estimator on the training set.

```
tree_reg1 = DecisionTreeRegressor(max_depth=2)
tree_reg1.fit(X, y)
```

- Now train the second estimator on the residual errors made by the first estimator.

```
y2 = y - tree_reg1.predict(X)
tree_reg2 = DecisionTreeRegressor(max_depth=2)
tree_reg2.fit(X, y2)
```

Gradient Boosting (3/3)

- Then we train the third estimator on the residual errors made by the second estimator.

```
y3 = y2 - tree_reg2.predict(X)
tree_reg3 = DecisionTreeRegressor(max_depth=2)
tree_reg3.fit(X, y3)
```

- Now we have an ensemble containing three trees.
- It can make predictions on a new instance simply by adding up the predictions of all the trees.

```
y_pred = sum(tree.predict(X_new) for tree in (tree_reg1, tree_reg2, tree_reg3))
```

Gradient Boosting in Spark (1/2)

- Two classes in `spark.ml`.
- Regression: `GBTRegressor`

```
val gbt = new GBTRegressor().setLabelCol("label").setFeaturesCol("features")
                             .setMaxIter(10).setFeatureSubsetStrategy("auto")

val model = gbt.fit(trainingData)
val predictions = model.transform(testData)
```

Gradient Boosting in Spark (1/2)

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val model = gbt.fit(trainingData)
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- Classifier: `GBTClassifier`

```
val gbt = new GBTClassifier().setLabelCol("label").setFeaturesCol("features")
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val predictions = model.transform(testData)
```

Summary

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- Decision tree
  - Top-down training algorithm
  - Termination condition
  - Feature selection: entropy, gini
- Ensemble models
  - Bagging: random forest
  - Boosting: AdaBoost, Gradient Boosting

Reference

- Aurélien Géron, Hands-On Machine Learning (Ch. 5, 6, 7)
- Matei Zaharia et al., Spark - The Definitive Guide (Ch. 27)

Questions?