Deep Feedforwards Networks

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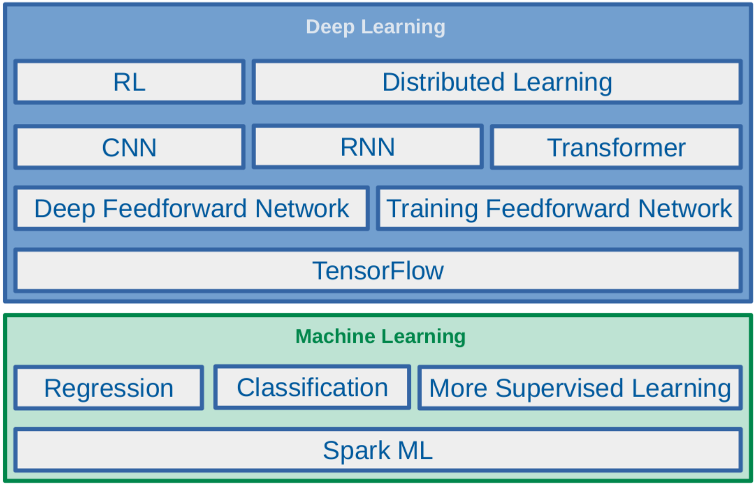
The Course Web Page

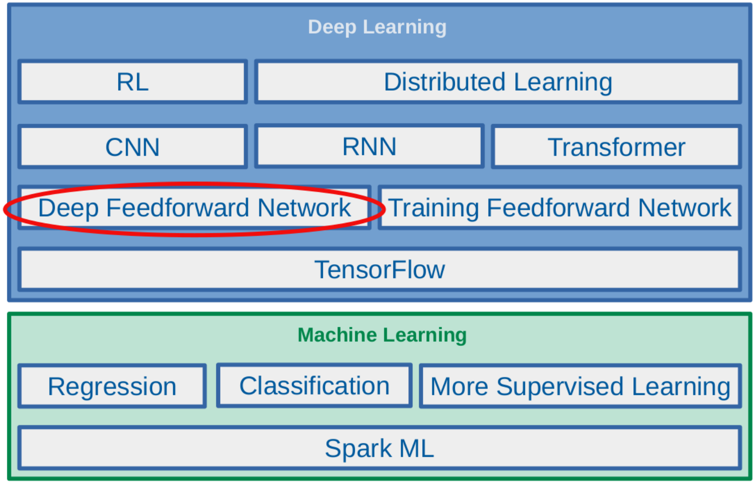
https://id2223kth.github.io

https://tinyurl.com/6s5jy46a

Where Are We?


Deep Learning
RL
Distributed Learning
CNN
RNN
Transformer
Deep Feedforward Network
Training Feedforward Network
TensorFlow
Machine Learning
Regression
Classification
More Supervised Learning
Spark ML

Where Are We?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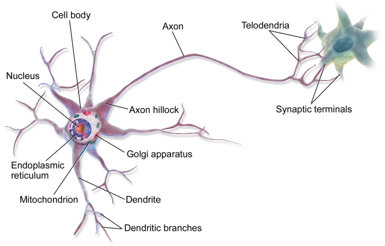
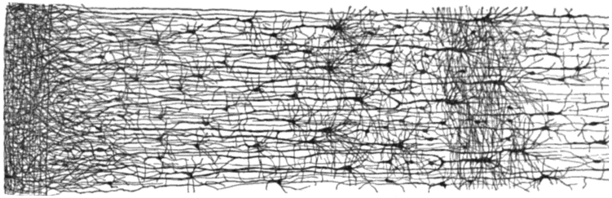
Biological Neurons (1/2)

- Brain architecture has inspired artificial neural networks.
- A biological neuron is composed of
  - Cell body, many dendrites (branching extensions), one axon (long extension), synapses
- Biological neurons receive signals from other neurons via these synapses.
- When a neuron receives a sufficient number of signals within a few milliseconds, it fires its own signals.



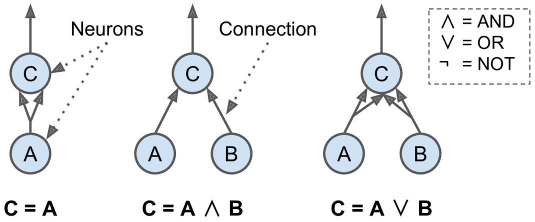
Biological Neurons (2/2)

- Biological neurons are organized in a vast network of billions of neurons.
- Each neuron typically is connected to thousands of other neurons.



A Simple Artificial Neural Network

- One or more binary inputs and one binary output
- Activates its output when more than a certain number of its inputs are active.

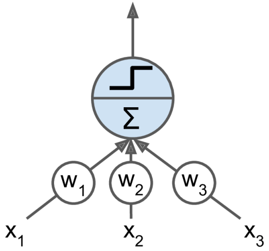


[A. Geron, O'Reilly Media, 2017]

The Linear Threshold Unit (LTU)

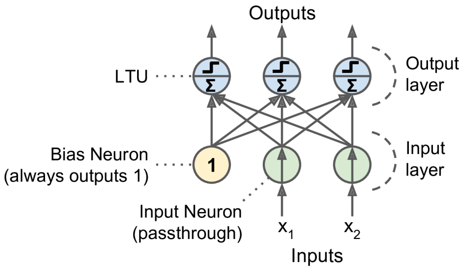
- Inputs of a LTU are numbers (not binary).
- Each input connection is associated with a weight.
- Computes a weighted sum of its inputs and applies a step function to that sum.

- $z = w_1x_1 + w_2x_2 + \cdots + w_nx_n = w^\intercal x$
- $\hat{y} = \text{step}(z) = \text{step}(w^\intercal x)$



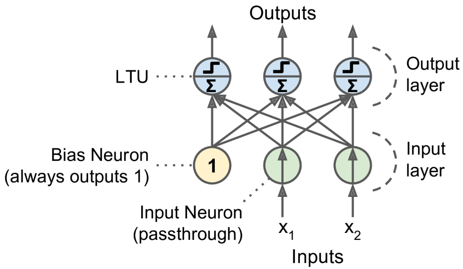
The Perceptron

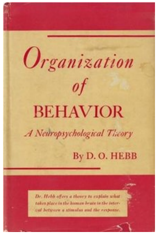
- The perceptron is a single layer of LTUs.
- The input neurons output whatever input they are fed.
- A bias neuron, which just outputs 1 all the time.
- If we use logistic function (sigmoid) instead of a step function, it computes a continuous output.



How is a Perceptron Trained? (1/2)

- The Perceptron training algorithm is inspired by Hebb's rule.
- When a biological neuron often triggers another neuron, the connection between these two neurons grows stronger.



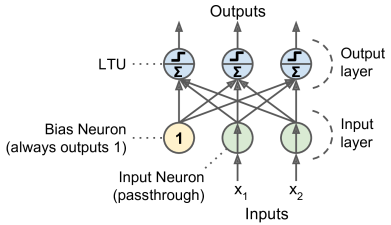


Organization of BEHAVIOR
A Neuropsychological Theory
By D. O. HEBB
Dr. Hebb offers a theory to explain what takes place in the human brain in the interval between a stimulus and the response.

How is a Perceptron Trained? (2/2)

- Feed one training instance $\mathtt{x}$ to each neuron $\mathtt{j}$ at a time and make its prediction $\hat{\mathtt{y}}$.
- Update the connection weights.

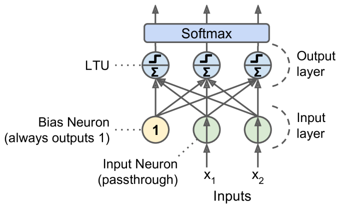
$$\hat{\mathtt{y}}_{\mathtt{j}} = \sigma(\mathbf{w}_{\mathtt{j}}^{\intercal}\mathbf{x} + \mathtt{b})$$
$$\mathtt{J}(\mathtt{w}_{\mathtt{j}}) = \mathtt{cross\_entropy}(\mathtt{y}_{\mathtt{j}}, \hat{\mathtt{y}}_{\mathtt{j}})$$
$$\mathtt{w}_{\mathtt{i,j}}^{(\mathtt{next})} = \mathtt{w}_{\mathtt{i,j}} - \eta\frac{\partial \mathtt{J}(\mathtt{w}_{\mathtt{j}})}{\mathtt{w}_{\mathtt{i}}}$$



- $\mathtt{w}_{\mathtt{i,j}}$: the weight between neurons $\mathtt{i}$ and $\mathtt{j}$.
- $\mathtt{x}_{\mathtt{i}}$: the $\mathtt{i}$th input value.
- $\hat{\mathtt{y}}_{\mathtt{j}}$: the $\mathtt{j}$th predicted output value.
- $\mathtt{y}_{\mathtt{j}}$: the $\mathtt{j}$th true output value.
- η: the learning rate.

Perceptron in TensorFlow

Perceptron in TensorFlow



```
n_neurons = 3
n_features = 2

model = keras.models.Sequential()
model.add(keras.layers.Dense(n_neurons, input_shape=(n_features,), activation="softmax"))
```

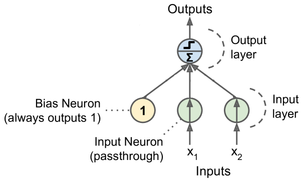
```
model.compile(loss="sparse_categorical_crossentropy", optimizer="sgd", metrics=["accuracy"])
model.fit(X_train, y_train, epochs=30)
```

Multi-Layer Perceptron (MLP)

Perceptron Weakness (1/2)

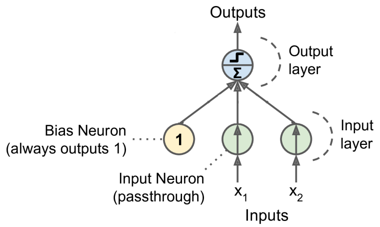
- Incapable of solving some trivial problems, e.g., XOR classification problem. Why?

A	B	A **XOR** B
0	0	0
0	1	1
1	0	1
1	1	0



$$X = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Perceptron Weakness (2/2)



$$X = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \qquad \hat{y} = \mathtt{step}(z), z = w_1 x_1 + w_2 x_2 + b$$

$$J(w) = \frac{1}{4} \sum_{x \in X} (\hat{y}(x) - y(x))^2$$

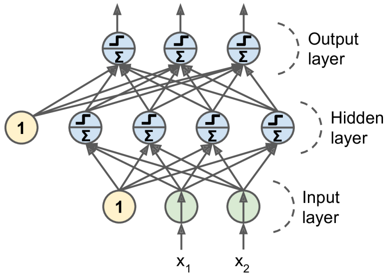
- If we minimize $J(w)$, we obtain $w_1 = 0$, $w_2 = 0$, and $b = \frac{1}{2}$.
- But, the model outputs 0.5 everywhere.

Multi-Layer Perceptron (MLP)

- The limitations of Perceptrons can be eliminated by stacking multiple Perceptrons.
- The resulting network is called a Multi-Layer Perceptron (MLP) or deep feedforward neural network.

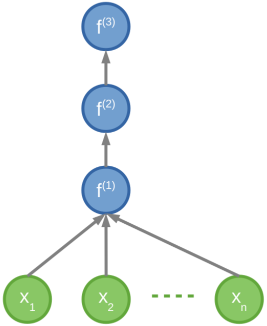
Feedforward Neural Network Architecture

- A feedforward neural network is composed of:
  - One input layer
  - One or more hidden layers
  - One final output layer
- Every layer except the output layer includes a bias neuron and is fully connected to the next layer.

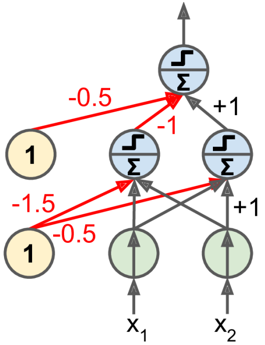


How Does it Work?

- The model is associated with a directed acyclic graph describing how the functions are composed together.
- E.g., assume a network with just a single neuron in each layer.
- Also assume we have three functions $f^{(1)}$, $f^{(2)}$, and $f^{(3)}$ connected in a chain: $\hat{y} = f(x) = f^{(3)}(f^{(2)}(f^{(1)}(x)))$
- $f^{(1)}$ is called the first layer of the network.
- $f^{(2)}$ is called the second layer, and so on.
- The length of the chain gives the depth of the model.

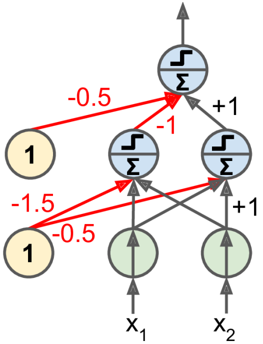


XOR with Feedforward Neural Network (1/3)



$$X = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \quad y = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad W_x = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad b_x = \begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix}$$

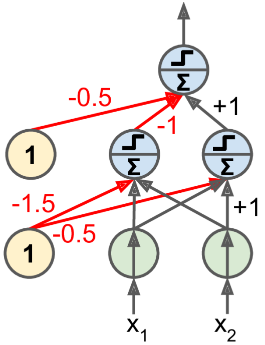
XOR with Feedforward Neural Network (2/3)



$$\mathrm{out_h} = XW_x^T + b_x = \begin{bmatrix} -1.5 & -0.5 \\ -0.5 & 0.5 \\ -0.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix} \qquad h = \mathtt{step}(\mathrm{out_h}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$w_h = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad b_h = -0.5$$

XOR with Feedforward Neural Network (3/3)



$$\text{out} = w_h^T h + b_h = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix} \qquad \text{step}(\text{out}) = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

How to Learn Model Parameters W?

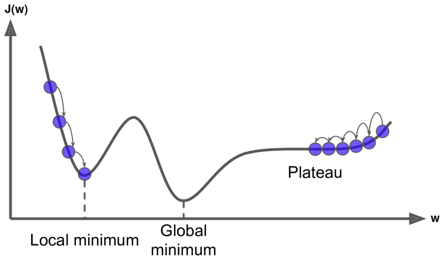
Feedforward Neural Network - Cost Function

- We use the cross-entropy (minimizing the negative log-likelihood) between the training data $\mathtt{y}$ and the model's predictions $\hat{\mathtt{y}}$ as the cost function.

$$\mathtt{cost}(\mathtt{y}, \hat{\mathtt{y}}) = -\sum_{\mathtt{j}} \mathtt{y}_{\mathtt{j}} \mathtt{log}(\hat{\mathtt{y}}_{\mathtt{j}})$$

Gradient-Based Learning (1/2)

- The most significant difference between the linear models we have seen so far and feedforward neural network?
- The non-linearity of a neural network causes its cost functions to become non-convex.
- Linear models, with convex cost function, guarantee to find global minimum.
  - Convex optimization converges starting from any initial parameters.

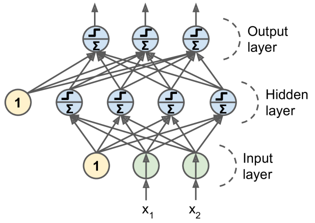


Gradient-Based Learning (2/2)

- Stochastic gradient descent applied to non-convex cost functions has no such convergence guarantee.
- It is sensitive to the values of the initial parameters.
- For feedforward neural networks, it is important to initialize all weights to small random values.
- The biases may be initialized to zero or to small positive values.

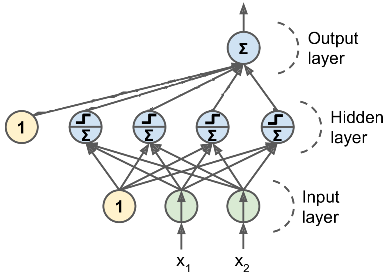
Training Feedforward Neural Networks

- How to train a feedforward neural network?
- For each training instance $x^{(i)}$ the algorithm does the following steps:
  1. Forward pass: make a prediction (compute $\hat{y}^{(i)} = f(x^{(i)})$).
  2. Measure the error (compute $\mathtt{cost}(\hat{y}^{(i)}, y^{(i)})$).
  3. Backward pass: go through each layer in reverse to measure the error contribution from each connection.
  4. Tweak the connection weights to reduce the error (update W and b).
- It's called the backpropagation training algorithm



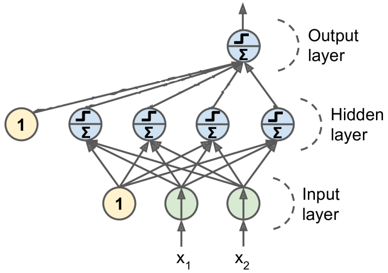
Output Unit (1/3)

- Linear units in neurons of the output layer.
- Output function: $\hat{y}_j = w_j^T h + b_j$.
- Cost function: minimizing the mean squared error.



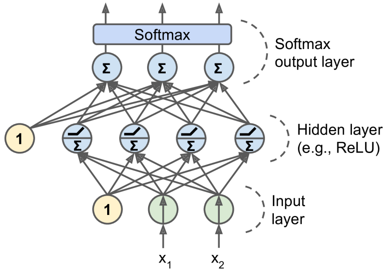
Output Unit (2/3)

- Sigmoid units in neurons of the output layer (binomial classification).
- Output function: $\hat{y}_j = \sigma(w_j^\intercal h + b_j)$.
- Cost function: minimizing the cross-entropy.



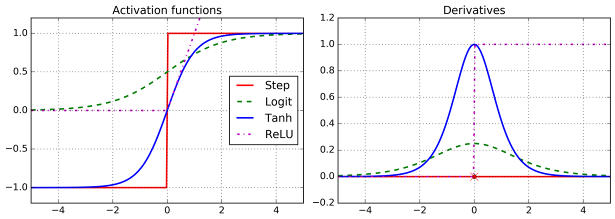
Output Unit (3/3)

- Softmax units in neurons of the output layer (multinomial classification).
- Output function: $\hat{y}_j = \texttt{softmax}(w_j^\intercal h + b_j)$.
- Cost function: minimizing the cross-entropy.



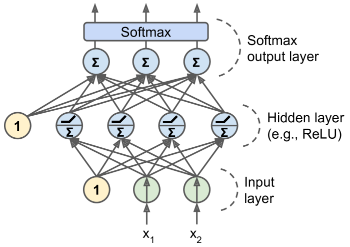
Hidden Units

- In order for the backpropagation algorithm to work properly, we need to replace the step function with other activation functions. Why?
- Alternative activation functions:
  1. Logistic function (sigmoid): $\sigma(z) = \frac{1}{1+e^{-z}}$
  2. Hyperbolic tangent function: $\mathtt{tanh}(z) = 2\sigma(2z) - 1$
  3. Rectified linear units (ReLUs): $\mathtt{ReLU}(z) = \max(0, z)$



Feedforward Network in TensorFlow

Feedforward Network in TensorFlow

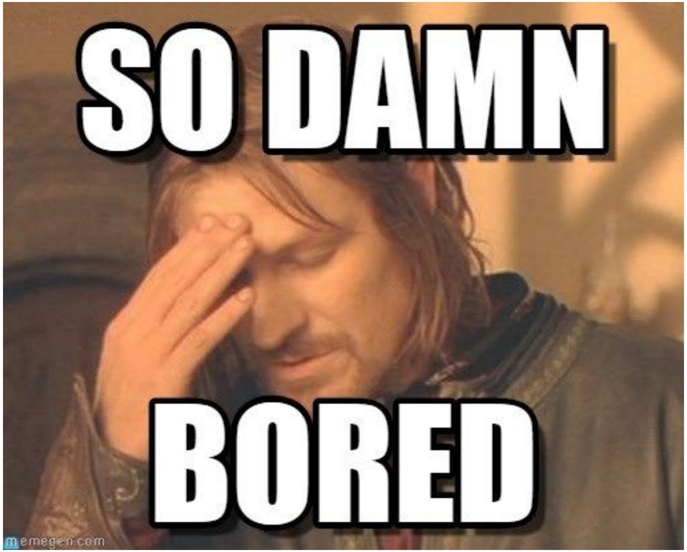


```
n_output = 3
n_hidden = 4
n_features = 2

model = keras.models.Sequential()
model.add(keras.layers.Dense(n_hidden, input_shape=(n_features,), activation="relu"))
model.add(keras.layers.Dense(n_output, activation="softmax"))
```

```
model.compile(loss="sparse_categorical_crossentropy", optimizer="sgd", metrics=["accuracy"])
model.fit(X_train, y_train, epochs=30)
```

Dive into Backpropagation Algorithm



[https://i.pinimg.com/originals/82/d9/2c/82d92c2c15c580c2b2fce65a83fe0b3f.jpg]

Chain Rule of Calculus (1/2)

- Assume $x \in \mathbb{R}$, and two functions f and g, and also assume $y = g(x)$ and $z = f(y) = f(g(x))$.
- The chain rule of calculus is used to compute the derivatives of functions, e.g., z, formed by composing other functions, e.g., g.
- Then the chain rule states that $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$
- Example:

$$z = f(y) = 5y^4 \text{ and } y = g(x) = x^3 + 7$$

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

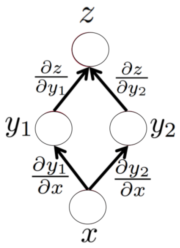
$$\frac{dz}{dy} = 20y^3 \text{ and } \frac{dy}{dx} = 3x^2$$

$$\frac{dz}{dx} = 20y^3 \times 3x^2 = 20(x^3 + 7) \times 3x^2$$

Chain Rule of Calculus (2/2)

- Two paths chain rule.

$$z = f(y_1, y_2) \text{ where } y_1 = g(x) \text{ and } y_2 = h(x)$$
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y_1}\frac{\partial y_1}{\partial x} + \frac{\partial z}{\partial y_2}\frac{\partial y_2}{\partial x}$$



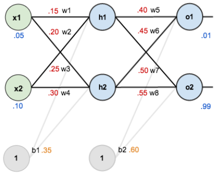
z
$\frac{\partial z}{\partial y_1}$
$\frac{\partial z}{\partial y_2}$
y_1
y_2
$\frac{\partial y_1}{\partial x}$
$\frac{\partial y_2}{\partial x}$
x

Backpropagation

- Backpropagation training algorithm for MLPs
- The algorithm repeats the following steps:
  1. Forward pass
  2. Backward pass

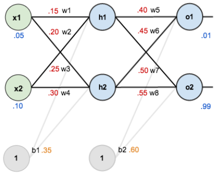
Backpropagation - Forward Pass

- Calculates outputs given input patterns.
- For each training instance
  - Feeds it to the network and computes the output of every neuron in each consecutive layer.
  - Measures the network's output error (i.e., the difference between the true and the predicted output of the network)
  - Computes how much each neuron in the last hidden layer contributed to each output neuron's error.



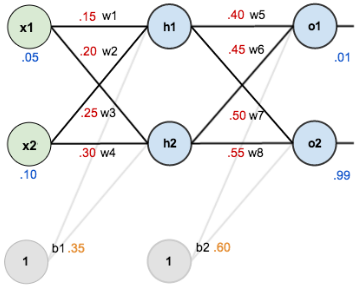
Backpropagation - Backward Pass

- Updates weights by calculating gradients.
- Measures how much of these error contributions came from each neuron in the previous hidden layer
  - Proceeds until the algorithm reaches the input layer.
- The last step is the gradient descent step on all the connection weights in the network, using the error gradients measured earlier.



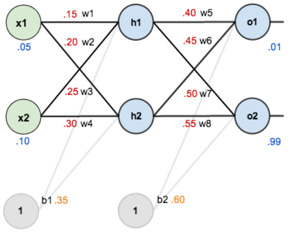
Backpropagation Example

- Two inputs, two hidden, and two output neurons.
- Bias in hidden and output neurons.
- Logistic activation in all the neurons.
- Squared error function as the cost function.



Backpropagation - Forward Pass (1/3)

- Compute the output of the hidden layer



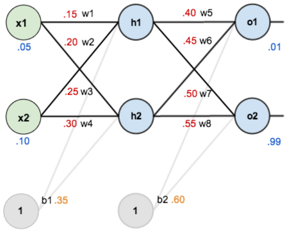
$$\mathtt{net}_{h1} = w_1 x_1 + w_2 x_2 + b_1 = 0.15 \times 0.05 + 0.2 \times 0.1 + 0.35 = 0.3775$$

$$\mathtt{out}_{h1} = \frac{1}{1 + e^{\mathtt{net}_{h1}}} = \frac{1}{1 + e^{0.3775}} = 0.59327$$

$$\mathtt{out}_{h2} = 0.59688$$

Backpropagation - Forward Pass (2/3)

- Compute the output of the output layer



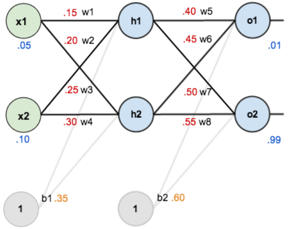
$$\mathtt{net}_{o1} = \mathtt{w}_5\mathtt{out}_{h1} + \mathtt{w}_6\mathtt{out}_{h2} + \mathtt{b}_2 = 0.4 \times 0.59327 + 0.45 \times 0.59688 + 0.6 = 1.1059$$

$$\mathtt{out}_{o1} = \frac{1}{1 + e^{\mathtt{net}_{o1}}} = \frac{1}{1 + e^{1.1059}} = 0.75136$$

$$\mathtt{out}_{o2} = 0.77292$$

Backpropagation - Forward Pass (3/3)

- Calculate the error for each output



$$\mathrm{E}_{o1} = \frac{1}{2}(\mathtt{target}_{o1} - \mathtt{output}_{o1})^2 = \frac{1}{2}(0.01 - 0.75136)^2 = 0.27481$$

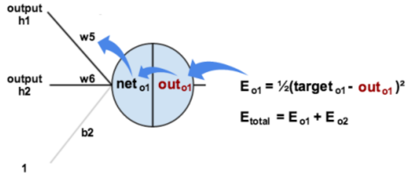
$$\mathrm{E}_{o2} = 0.02356$$

$$\mathrm{E}_{\mathtt{total}} = \sum \frac{1}{2}(\mathtt{target} - \mathtt{output})^2 = \mathrm{E}_{o1} + \mathrm{E}_{o2} = 0.27481 + 0.02356 = 0.29837$$

[http://marimancusi.blogspot.com/2015/09/are-you-book-dragon.html]

Backpropagation - Backward Pass - Output Layer (1/6)

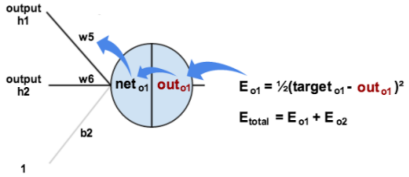
- Consider $\mathtt{w}_5$
- We want to know how much a change in $\mathtt{w}_5$ affects the total error $\left(\frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{w}_5}\right)$
- Applying the chain rule



$$\frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{w}_5} = \frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{out}_{\mathtt{o1}}} \times \frac{\partial \mathtt{out}_{\mathtt{o1}}}{\partial \mathtt{net}_{\mathtt{o1}}} \times \frac{\partial \mathtt{net}_{\mathtt{o1}}}{\partial \mathtt{w}_5}$$

Backpropagation - Backward Pass - Output Layer (2/6)

- First, how much does the total error change with respect to the output? ($\frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{out}_{\mathtt{o1}}}$)



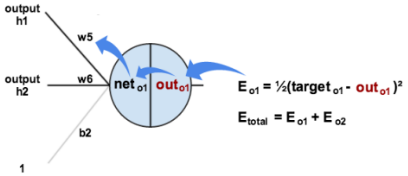
$$\frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{w}_5} = \frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{out}_{\mathtt{o1}}} \times \frac{\partial \mathtt{out}_{\mathtt{o1}}}{\partial \mathtt{net}_{\mathtt{o1}}} \times \frac{\partial \mathtt{net}_{\mathtt{o1}}}{\partial \mathtt{w}_5}$$

$$\mathtt{E}_{\mathtt{total}} = \frac{1}{2}(\mathtt{target}_{\mathtt{o1}} - \mathtt{out}_{\mathtt{o1}})^2 + \frac{1}{2}(\mathtt{target}_{\mathtt{o2}} - \mathtt{out}_{\mathtt{o2}})^2$$

$$\frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{out}_{\mathtt{o1}}} = -2\frac{1}{2}(\mathtt{target}_{\mathtt{o1}} - \mathtt{out}_{\mathtt{o1}}) = -(0.01 - 0.75136) = 0.74136$$

Backpropagation - Backward Pass - Output Layer (3/6)

- Next, how much does the $\mathtt{out}_{o1}$ change with respect to its total input $\mathtt{net}_{o1}$? $\left(\frac{\partial \mathtt{out}_{o1}}{\partial \mathtt{net}_{o1}}\right)$



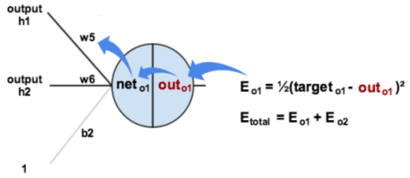
$$\frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{w}_5} = \frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{out}_{o1}} \times \frac{\partial \mathtt{out}_{o1}}{\partial \mathtt{net}_{o1}} \times \frac{\partial \mathtt{net}_{o1}}{\partial \mathtt{w}_5}$$

$$\mathtt{out}_{o1} = \frac{1}{1 + e^{-\mathtt{net}_{o1}}}$$

$$\frac{\partial \mathtt{out}_{o1}}{\partial \mathtt{net}_{o1}} = \mathtt{out}_{o1}(1 - \mathtt{out}_{o1}) = 0.75136(1 - 0.75136) = 0.18681$$

Backpropagation - Backward Pass - Output Layer (4/6)

- Finally, how much does the total $\mathtt{net}_{o1}$ change with respect to $\mathtt{w}_5$? ($\frac{\partial \mathtt{net}_{o1}}{\partial \mathtt{w}_5}$)

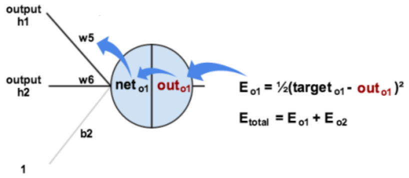


$$\frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{w}_5} = \frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{out}_{o1}} \times \frac{\partial \mathtt{out}_{o1}}{\partial \mathtt{net}_{o1}} \times \frac{\partial \mathtt{net}_{o1}}{\partial \mathtt{w}_5}$$

$$\mathtt{net}_{o1} = \mathtt{w}_5 \times \mathtt{out}_{h1} + \mathtt{w}_6 \times \mathtt{out}_{h2} + \mathtt{b}_2$$
$$\frac{\partial \mathtt{net}_{o1}}{\partial \mathtt{w}_5} = \mathtt{out}_{h1} = 0.59327$$

Backpropagation - Backward Pass - Output Layer (5/6)

- Putting it all together:



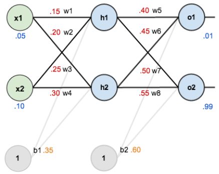
output h1
w5
output h2
w6
b2
1
net o1
out o1
$E_{o1} = \frac{1}{2}(target_{o1} - out_{o1})^2$
$E_{total} = E_{o1} + E_{o2}$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} \times \frac{\partial out_{o1}}{\partial net_{o1}} \times \frac{\partial net_{o1}}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial w_5} = 0.74136 \times 0.18681 \times 0.59327 = 0.08216$$

Backpropagation - Backward Pass - Output Layer (6/6)

- To decrease the error, we subtract this value from the current weight.
- We assume that the learning rate is $\eta = 0.5$.

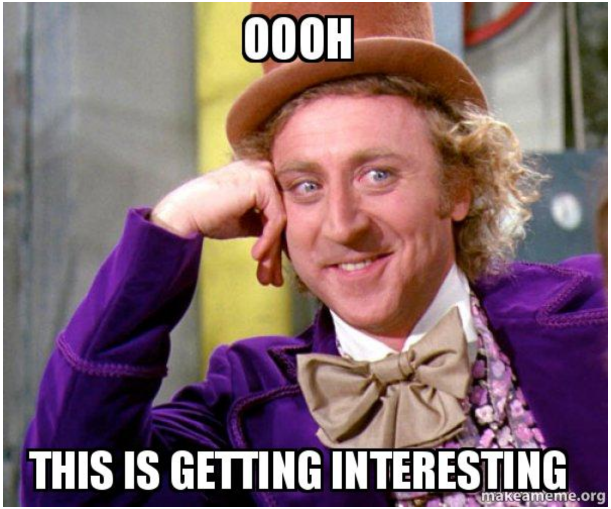


$$\mathtt{w}_5^{(\mathtt{next})} = \mathtt{w}_5 - \eta \times \frac{\partial \mathrm{E}_{\mathtt{total}}}{\partial \mathtt{w}_5} = 0.4 - 0.5 \times 0.08216 = 0.35891$$

$$\mathtt{w}_6^{(\mathtt{next})} = 0.40866$$

$$\mathtt{w}_7^{(\mathtt{next})} = 0.5113$$

$$\mathtt{w}_8^{(\mathtt{next})} = 0.56137$$

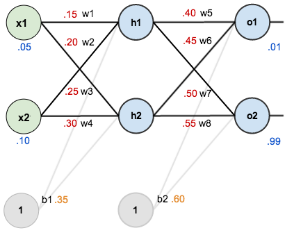


[https://makeameme.org/meme/oooh-this]

Backpropagation - Backward Pass - Hidden Layer (1/8)

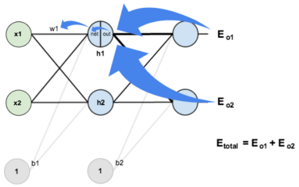
- Continue the backwards pass by calculating new values for w_1, w_2, w_3, and w_4.
- For w_1 we have:

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} \times \frac{\partial out_{h1}}{\partial net_{h1}} \times \frac{\partial net_{h1}}{\partial w_1}$$



Backpropagation - Backward Pass - Hidden Layer (2/8)

- Here, the output of each hidden layer neuron contributes to the output of multiple output neurons.
- E.g., $\mathtt{out}_{h1}$ affects both $\mathtt{out}_{o1}$ and $\mathtt{out}_{o2}$, so $\frac{\partial E_{total}}{\partial \mathtt{out}_{h1}}$ needs to take into consideration its effect on the both output neurons.

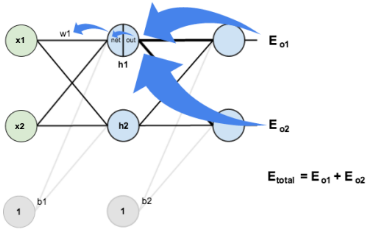


$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial \mathtt{out}_{h1}} \times \frac{\partial \mathtt{out}_{h1}}{\partial \mathtt{net}_{h1}} \times \frac{\partial \mathtt{net}_{h1}}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial \mathtt{out}_{h1}} = \frac{\partial E_{o1}}{\partial \mathtt{out}_{h1}} + \frac{\partial E_{o2}}{\partial \mathtt{out}_{h1}}$$

Backpropagation - Backward Pass - Hidden Layer (3/8)

- Starting with $\frac{\partial E_{o1}}{\partial out_{h1}}$



$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}}$$

$$\frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{o1}} \times \frac{\partial out_{o1}}{\partial net_{o1}} \times \frac{\partial net_{o1}}{\partial out_{h1}}$$

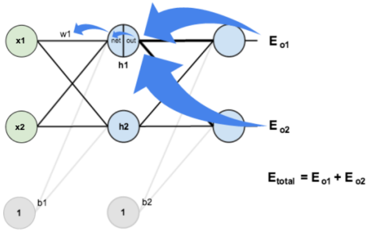
$$\frac{\partial E_{o1}}{\partial out_{o1}} = 0.74136, \frac{\partial out_{o1}}{\partial net_{o1}} = 0.18681$$

$$net_{o1} = w_5 \times out_{h1} + w_6 \times out_{h2} + b_2$$

$$\frac{\partial net_{o1}}{\partial out_{h1}} = w_5 = 0.40$$

Backpropagation - Backward Pass - Hidden Layer (4/8)

- Plugging them together.



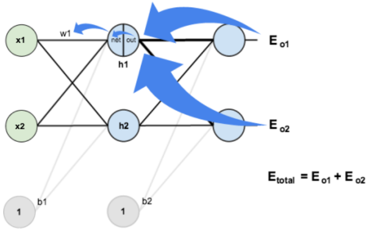
$$\frac{\partial E_{o1}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{o1}} \times \frac{\partial out_{o1}}{\partial net_{o1}} \times \frac{\partial net_{o1}}{\partial out_{h1}} = 0.74136 \times 0.18681 \times 0.40 = 0.0554$$

$$\frac{\partial E_{o2}}{\partial out_{h1}} = -0.01905$$

$$\frac{\partial E_{total}}{\partial out_{h1}} = \frac{\partial E_{o1}}{\partial out_{h1}} + \frac{\partial E_{o2}}{\partial out_{h1}} = 0.0554 + -0.01905 = 0.03635$$

Backpropagation - Backward Pass - Hidden Layer (5/8)

- Now we need to figure out $\frac{\partial \mathtt{out}_{\mathtt{h1}}}{\partial \mathtt{net}_{\mathtt{h1}}}$.



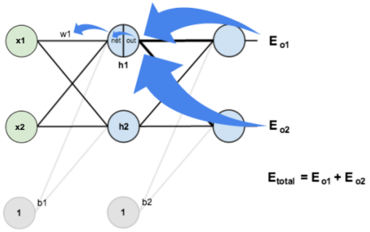
$$\frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{w}_1} = \frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{out}_{\mathtt{h1}}} \times \frac{\partial \mathtt{out}_{\mathtt{h1}}}{\partial \mathtt{net}_{\mathtt{h1}}} \times \frac{\partial \mathtt{net}_{\mathtt{h1}}}{\partial \mathtt{w}_1}$$

$$\mathtt{out}_{\mathtt{h1}} = \frac{1}{1 + \mathtt{e}^{-\mathtt{net}_{\mathtt{h1}}}}$$

$$\frac{\partial \mathtt{out}_{\mathtt{h1}}}{\partial \mathtt{net}_{\mathtt{h1}}} = \mathtt{out}_{\mathtt{h1}}(1 - \mathtt{out}_{\mathtt{h1}}) = 0.59327(1 - 0.59327) = 0.2413$$

Backpropagation - Backward Pass - Hidden Layer (6/8)

- And then $\frac{\partial \mathtt{net}_{h1}}{\partial \mathtt{w}_1}$.



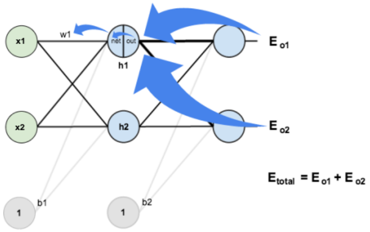
$$\frac{\partial \mathtt{E}_{total}}{\partial \mathtt{w}_1} = \frac{\partial \mathtt{E}_{total}}{\partial \mathtt{out}_{h1}} \times \frac{\partial \mathtt{out}_{h1}}{\partial \mathtt{net}_{h1}} \times \frac{\partial \mathtt{net}_{h1}}{\partial \mathtt{w}_1}$$

$$\mathtt{net}_{h1} = \mathtt{w}_1 \mathtt{x}_1 + \mathtt{w}_2 \mathtt{x}_2 + \mathtt{b}_1$$

$$\frac{\partial \mathtt{net}_{h1}}{\partial \mathtt{w}_1} = \mathtt{x}_1 = 0.05$$

Backpropagation - Backward Pass - Hidden Layer (7/8)

- Putting it all together.

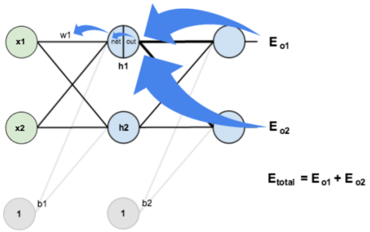


$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{h1}} \times \frac{\partial out_{h1}}{\partial net_{h1}} \times \frac{\partial net_{h1}}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial w_1} = 0.03635 \times 0.2413 \times 0.05 = 0.00043$$

Backpropagation - Backward Pass - Hidden Layer (8/8)

- We can now update $\mathtt{w}_1$.
- Repeating this for $\mathtt{w}_2$, $\mathtt{w}_3$, and $\mathtt{w}_4$.



$$\mathtt{w}_1^{(\mathtt{next})} = \mathtt{w}_1 - \eta \times \frac{\partial \mathtt{E}_{\mathtt{total}}}{\partial \mathtt{w}_1} = 0.15 - 0.5 \times 0.00043 = 0.14978$$

$$\mathtt{w}_2^{(\mathtt{next})} = 0.19956$$

$$\mathtt{w}_3^{(\mathtt{next})} = 0.24975$$

$$\mathtt{w}_4^{(\mathtt{next})} = 0.2995$$

Summary

Summary

- LTU
- Perceptron
- Perceptron weakness
- MLP and feedforward neural network
- Gradient-based learning
- Backpropagation: forward pass and backward pass
- Output unit: linear, sigmoid, softmax
- Hidden units: sigmoid, tanh, relu

Reference

- Ian Goodfellow et al., Deep Learning (Ch. 6)
- Aurélien Géron, Hands-On Machine Learning (Ch. 10)

Questions?