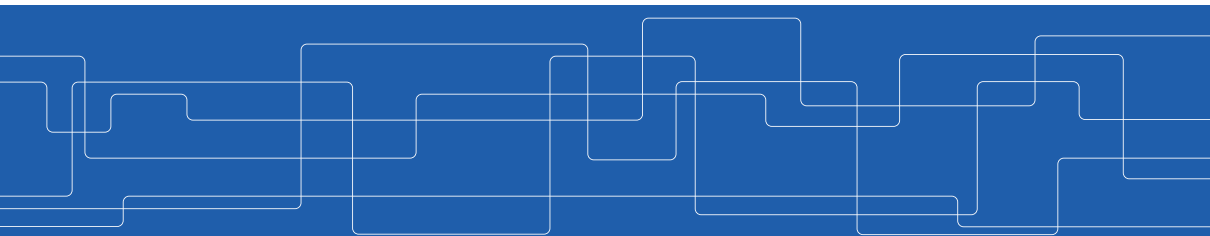




Training Deep Feedforwards Networks

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2021-11-24



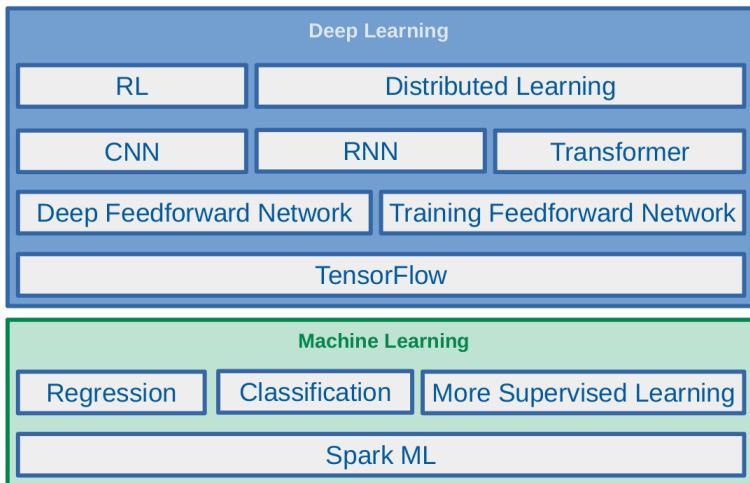


The Course Web Page

`https://id2223kth.github.io`
`https://tinyurl.com/6s5jy46a`

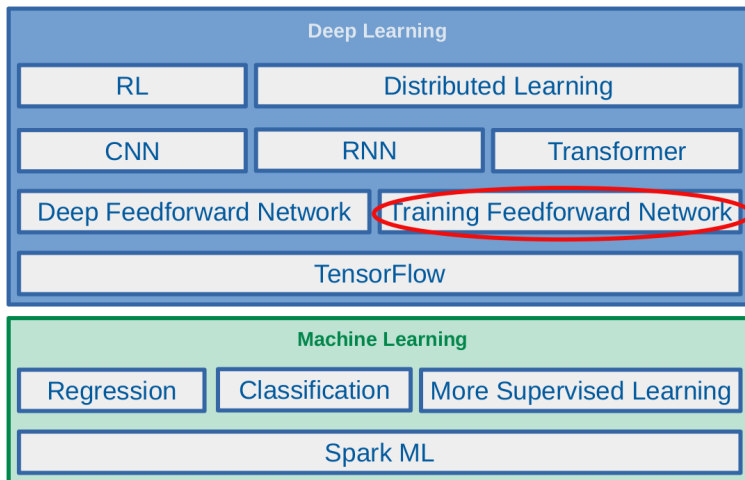


Where Are We?



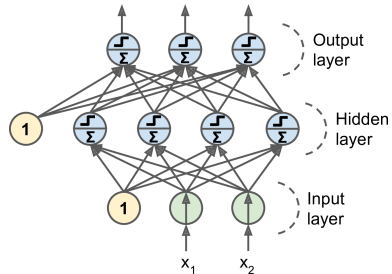


Where Are We?

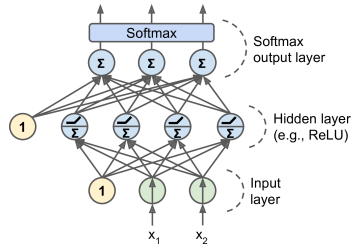


Feedforward Neural Network Architecture

- A **feedforward neural network** is composed of:
- One **input layer**
 - One or more **hidden layers**
 - One final **output layer**



Feedforward Network in TensorFlow



```
n_output = 3
n_hidden = 4
n_features = 2
```

```
model = keras.models.Sequential()
model.add(keras.layers.Dense(n_hidden, input_shape=(n_features,), activation="relu"))
model.add(keras.layers.Dense(n_output, activation="softmax"))
```

```
model.compile(loss="sparse_categorical_crossentropy", optimizer="sgd", metrics=["accuracy"])
model.fit(X_train, y_train, epochs=30)
```

Challenges of Training Feedforward Neural Networks

- ▶ Challenges ...
- ▶ **Overfitting**: risk of **overfitting** a model with **large number** of parameters.
- ▶ **Vanishing/exploding gradients**: hard to train **lower layers**.
- ▶ **Training speed**: **slow training** with large networks.



Overfitting



High Degree of Freedom and Overfitting Problem

- ▶ With large number of parameters, a network has a high degree of freedom.
- ▶ It can fit a huge variety of complex datasets.
- ▶ This flexibility also means that it is prone to overfitting on training set.
- ▶ Let's reduce the degree of freedom a model.



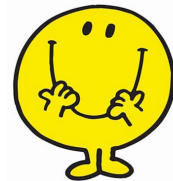
Avoiding Overfitting

- ▶ Early stopping
- ▶ l_1 and l_2 regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation



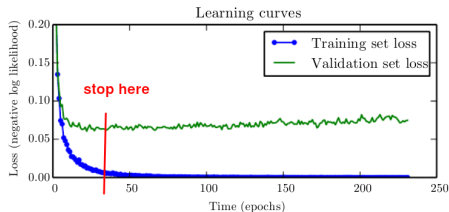
Avoiding Overfitting

- ▶ **Early stopping**
- ▶ l_1 and l_2 regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation



Early Stopping (1/2)

- ▶ As the **training steps go by**, its **prediction error on the training/validation set naturally goes down**.
- ▶ After a while the **validation error stops decreasing** and **starts to go back up**.
 - The model has started to **overfit the training data**.
- ▶ In the **early stopping**, we **stop training** when the **validation error reaches a minimum**.





Early Stopping (2/2)

```
from tensorflow.keras.callbacks import EarlyStopping  
  
model = tf.keras.models.Sequential(...)  
  
model.compile(optimizer='sgd', loss='sparse_categorical_crossentropy', metrics=['accuracy'])  
  
earlystop_callback = EarlyStopping(monitor='accuracy', min_delta=0.05, patience=1)  
  
model.fit(x_train, y_train, epochs=500, callbacks=[earlystop_callback])
```

Avoiding Overfitting

- ▶ Early stopping
- ▶ l_1 and l_2 regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation





l_1 and l_2 Regularization (1/3)

- ▶ Penalize large values of weights w_j .

$$\tilde{J}(w) = J(w) + \lambda R(w)$$

- ▶ Two questions:
 1. How should we define $R(w)$?
 2. How do we determine λ ?



l_1 and l_2 Regularization (2/3)

- ▶ l_1 regression: $R(w) = \lambda \sum_{i=1}^n |w_i|$ is added to the **cost function**.

$$\tilde{J}(w) = J(w) + \lambda \sum_{i=1}^n |w_i|$$

```
keras.layers.Dense(100, activation="relu", kernel_regularizer=keras.regularizers.l1(0.1))
```




l_1 and l_2 Regularization (3/3)

- ▶ l_2 regression: $R(w) = \lambda \sum_{i=1}^n w_i^2$ is added to the cost function.

$$\tilde{J}(w) = J(w) + \lambda \sum_{i=1}^n w_i^2$$

```
keras.layers.Dense(100, activation="relu", kernel_regularizer=keras.regularizers.l2(0.01))
```

Avoiding Overfitting

- ▶ Early stopping
- ▶ l_1 and l_2 regularization
- ▶ **Max-norm regularization**
- ▶ Dropout
- ▶ Data augmentation





Max-Norm Regularization

- ▶ **Max-norm regularization**: constrains the weights w_j of the incoming connections for each neuron j .
 - Prevents them from getting too large.

- ▶ After each training step, clip w_j as below, if $\|w_j\|_2 > r$:

$$w_j \leftarrow w_j \frac{r}{\|w_j\|_2}$$

- r is the max-norm hyperparameter

- $\|w_j\|_2 = (\sum_i w_{i,j}^2)^{\frac{1}{2}} = \sqrt{w_{1,j}^2 + w_{2,j}^2 + \dots + w_{n,j}^2}$

```
keras.layers.Dense(100, activation="relu", kernel_constraint=keras.constraints.max_norm(1.))
```

Avoiding Overfitting

- ▶ Early stopping
- ▶ l_1 and l_2 regularization
- ▶ Max-norm regularization
- ▶ **Dropout**
- ▶ Data augmentation



Dropout (1/4)

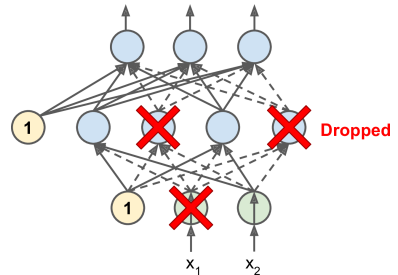
- ▶ Would a **company** perform better if its employees were told to **toss a coin** every morning to decide **whether or not to go to work**?



Dropout (2/4)

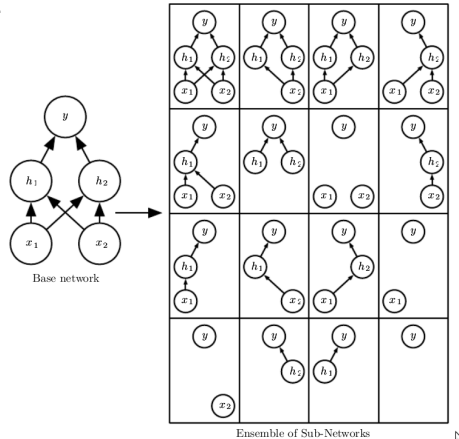
- ▶ At each **training step**, each neuron drops out temporarily with a **probability p** .
 - The **hyperparameter p** is called the **dropout rate**.
 - A neuron will be **entirely ignored** during **this training step**.
 - It may be **active** during the **next step**.
 - Exclude the **output neurons**.

- ▶ **After training**, neurons **don't get dropped** anymore.



Dropout (3/4)

- ▶ Each neuron can be either **present or absent**.
- ▶ 2^N possible networks, where N is the total number of **droppable neurons**.
 - $N = 4$ in this figure.





Dropout (4/4)

```
model = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.Dropout(rate=0.2),
    keras.layers.Dense(128, activation="relu"),
    keras.layers.Dropout(rate=0.2),
    keras.layers.Dense(10, activation="softmax")
])
```

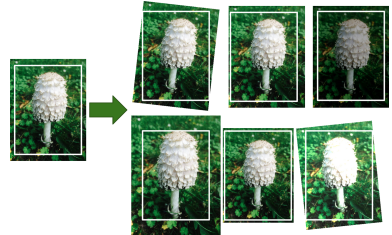

Avoiding Overfitting

- ▶ Early stopping
- ▶ l_1 and l_2 regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ **Data augmentation**



Data Augmentation

- ▶ One way to make a model **generalize better** is to **train it on more data**.
- ▶ This will **reduce overfitting**.
- ▶ Create **fake data** and add it to the **training set**.
 - E.g., in an **image classification** we can slightly shift, rotate and resize an image.
 - **Add the resulting pictures** to the **training set**.



Vanishing/Exploding Gradients





Vanishing/Exploding Gradients Problem (1/4)

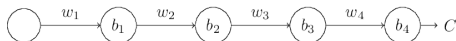
- ▶ The **backpropagation** goes from **output to input** layer, and propagates the **error gradient** on the way.

$$w^{(\text{next})} = w - \eta \frac{\partial J(w)}{\partial w}$$

- ▶ Gradients often get **smaller and smaller** as the algorithm progresses **down to the lower layers**.
- ▶ As a result, the gradient descent update leaves the **lower layer connection weights** virtually **unchanged**.
- ▶ This is called the **vanishing gradients** problem.

Vanishing/Exploding Gradients Problem (2/4)

- ▶ Assume a network with just a **single neuron** in **each layer**.



- w_1, w_2, \dots are the **weights**
 - b_1, b_2, \dots are the **biases**
 - C is the **cost function**
- ▶ The output a_j from the j th neuron is $\sigma(z_j)$.
 - σ is the **sigmoid** activation function
 - $z_j = w_j a_{j-1} + b_j$
 - E.g., $a_4 = \sigma(z_4) = \text{sigmoid}(w_4 a_3 + b_4)$

Vanishing/Exploding Gradients Problem (3/4)

- Lets compute the **gradient** associated to the **first hidden neuron** ($\frac{\partial C}{\partial b_1}$).



$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times \frac{\partial z_4}{\partial a_3} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial z_3}{\partial a_2} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial z_2}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial z_1}{\partial b_1}$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times \frac{\partial w_4 a_3 + b_4}{\partial a_3} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial w_3 a_2 + b_3}{\partial a_2} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial w_2 a_1 + b_2}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial w_1 a_0 + b_1}{\partial b_1}$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times w_4 \times \frac{\partial a_3}{\partial z_3} \times w_3 \times \frac{\partial a_2}{\partial z_2} \times w_2 \times \frac{\partial a_1}{\partial z_1} \times 1$$

Vanishing/Exploding Gradients Problem (4/4)

- Now, consider $\frac{\partial C}{\partial b_3}$.



$$\frac{\partial C}{\partial b_3} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times w_4 \times \frac{\partial a_3}{\partial z_3}$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times w_4 \times \frac{\partial a_3}{\partial z_3} \times w_3 \times \frac{\partial a_2}{\partial z_2} \times w_2 \times \frac{\partial a_1}{\partial z_1} \times 1$$

- Assume $w_3 \times \frac{\partial a_2}{\partial z_2} < \frac{1}{4}$ and $w_2 \times \frac{\partial a_1}{\partial z_1} < \frac{1}{4}$
- The gradient $\frac{\partial C}{\partial b_1}$ be a factor of 16 (or more) smaller than $\frac{\partial C}{\partial b_3}$.
 - This is the essential **origin** of the **vanishing gradient problem**.

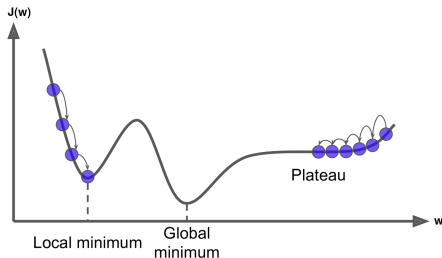
Overcoming the Vanishing Gradient

- ▶ **Parameter initialization strategies**
- ▶ Nonsaturating activation function
- ▶ Batch normalization
- ▶ Gradient clipping



Parameter Initialization Strategies (1/4)

- ▶ The **non-linearity** of a neural network causes the **cost functions** to become **non-convex**.
- ▶ The stochastic gradient descent on **non-convex cost functions** performs is **sensitive** to the values of the **initial parameters**.
- ▶ Designing initialization strategies is a **difficult task**.





Parameter Initialization Strategies (2/4)

- ▶ The **initial parameters** need to **break symmetry** between **different units**.
- ▶ **Two hidden units** with the **same activation function** connected to the **same inputs**, must have **different** initial parameters.
 - The goal of having each unit **compute a different function**.
- ▶ It motivates **random initialization** of the parameters.
 - Typically, we set the **biases** to **constants**, and initialize only the **weights randomly**.



Parameter Initialization Strategies (3/4)

- ▶ We need the signals to flow properly in both directions.
- ▶ The Glorot and Bengio initialization proposed that:
 - The variance of the outputs of each layer to be equal to the variance of its inputs.
 - The gradients to have equal variance before and after flowing through a layer in the reverse direction.
- ▶ It is not possible to guarantee both unless each layer has an equal number of inputs and neurons.
- ▶ Based on the Xavier initialization, the weights are initialized using normal distribution with mean 0 and the following standard deviation.



Parameter Initialization Strategies (4/4)

- ▶ fan_{in} and fan_{out} are the **number of inputs and neurons** for the layer whose weights are being initialized.
- ▶ $\text{fan}_{\text{avg}} = \frac{2}{\text{fan}_{\text{in}} + \text{fan}_{\text{out}}}$
- ▶ **Glorot** initialization, for **none**, **logistic**, **sigmoid**, and **tanh**: $\sigma^2 = \frac{1}{\text{fan}_{\text{avg}}}$
- ▶ **He** initialization, for **ReLU**: $\sigma^2 = \frac{2}{\text{fan}_{\text{in}}}$

```
keras.layers.Dense(10, activation="relu", kernel_initializer="he_normal")
```

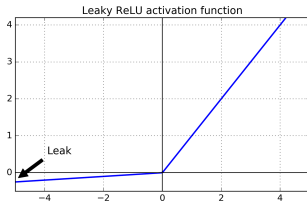
Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ **Nonsaturating activation function**
- ▶ Batch normalization
- ▶ Gradient clipping



Nonsaturating Activation Functions (1/4)

- ▶ $\text{ReLU}(z) = \max(0, z)$
- ▶ The **dying ReLUs** problem.
 - During **training**, some neurons **stop outputting anything other than 0**.
 - E.g., when the **weighted sum of the neuron's inputs is negative**, it starts outputting 0.
- ▶ Use **leaky ReLU** instead: $\text{LeakyReLU}_\alpha(z) = \max(\alpha z, z)$.
 - α is the **slope** of the function for $z < 0$.



Nonsaturating Activation Functions (2/4)

▶ Randomized Leaky ReLU (RRReLU)

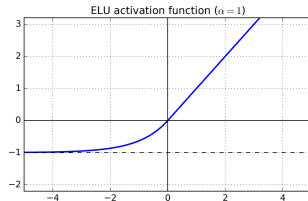
- α is picked randomly during training, and it is fixed during testing.

▶ Parametric Leaky ReLU (PReLU)

- Learn α during training (instead of being a hyperparameter).

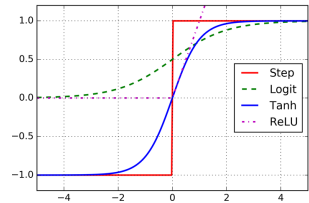
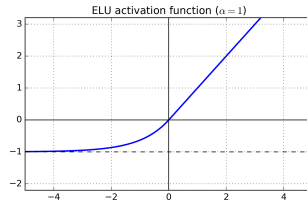
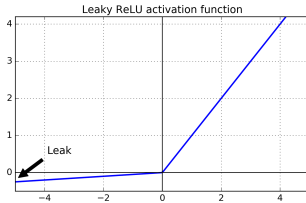
▶ Exponential Linear Unit (ELU)

$$\text{ELU}_{\alpha}(z) = \begin{cases} \alpha(\exp(z) - 1) & \text{if } z < 0 \\ z & \text{if } z \geq 0 \end{cases}$$



Nonsaturating Activation Functions (3/4)

- ▶ Which activation function should we use?
- ▶ In general $\text{logistic} < \text{tanh} < \text{ReLU} < \text{leaky ReLU (and its variants)} < \text{ELU}$
- ▶ If you care about runtime performance, then leaky ReLUs works better than ELUs.





Nonsaturating Activation Functions (4/4)

```
# elu  
keras.layers.Dense(10, activation="elu")
```

```
# leaky relu  
model = keras.models.Sequential([  
    keras.layers.Flatten(input_shape=[28, 28]),  
    keras.layers.Dense(128, kernel_initializer="he_normal"),  
    keras.layers.LeakyReLU(),  
    keras.layers.Dense(10, activation="softmax")  
])
```

Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ **Batch normalization**
- ▶ Gradient clipping





Batch Normalization (1/4)

- ▶ The gradient tells how to **update each parameter**, under the assumption that **the other layers do not change**.
 - In practice, we update all of the layers **simultaneously**.
 - However, **unexpected results can happen**.
- ▶ **Batch normalization** makes the **learning of layers** in the network more **independent of each other**.
 - It is a technique to address the problem that the **distribution of each layer's inputs** changes **during training**, as the parameters of the **previous layers change**.
- ▶ The technique consists of **adding an operation** in the model just **before the activation function** of each layer.



Batch Normalization (2/4)

- ▶ It's **zero-centering** and **normalizing the inputs**, then **scaling** and **shifting the result**.
 - Estimates the **inputs' mean** and **standard deviation** of the **current mini-batch**.

$$\mu_B = \frac{1}{m_B} \sum_{i=1}^{m_B} x^{(i)}$$

$$\sigma_B^2 = \frac{1}{m_B} \sum_{i=1}^{m_B} (x^{(i)} - \mu_B)^2$$

- ▶ μ_B : the **empirical mean**, evaluated over the whole **mini-batch B**.
- ▶ σ_B : the **empirical standard deviation**, also evaluated over the whole **mini-batch**.
- ▶ m_B : the **number of instances** in the mini-batch.



Batch Normalization (3/4)

$$\hat{x}^{(i)} = \frac{x^{(i)} - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$
$$z^{(i)} = \gamma \hat{x}^{(i)} + \beta$$

- ▶ $\hat{x}^{(i)}$: the **zero-centered and normalized input**.
- ▶ $z^{(i)}$: the output of the **BN operation**, which is a scaled and shifted version of the inputs.
- ▶ γ : the **scaling parameter** vector for the layer.
- ▶ β : the **shifting parameter (offset)** vector for the layer.
- ▶ ϵ : a tiny number to **avoid division by zero**.
- ▶ \otimes : represents the **element-wise multiplication**.



Batch Normalization (4/4)

```
model = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.BatchNormalization(),
    keras.layers.Dense(128, activation="relu"),
    keras.layers.BatchNormalization(),
    keras.layers.Dense(10, activation="softmax")
])
```

Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ Batch normalization
- ▶ Gradient clipping





Gradient Clipping

- ▶ **Gradient clipping**: clip the gradients during **backpropagation** so that they **never exceed** some threshold.

```
optimizer = keras.optimizers.SGD(clipvalue=1.0)
model.compile(loss="mse", optimizer=optimizer)
```

- ▶ Setting the **clipvalue** or **clipnorm** argument when creating an optimizer.
- ▶ **clipvalue=1.0** and **clipnorm=1.0**: values between -1.0 and 1.0.
- ▶ **clipvalue=1.0**: $[0.9, 100.0] \Rightarrow [0.9, 1.0]$
- ▶ **clipnorm=1.0**: $[0.9, 100.0] \Rightarrow [0.00899964, 0.9999595]$

Training Speed

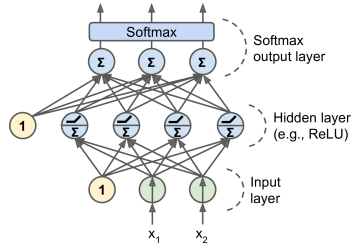




Regular Gradient Descent Optimization (1/2)

- ▶ Gradient descent optimization algorithm
- ▶ It updates the weights $w_i^{(\text{next})} = w_i - \eta \frac{\partial J(w)}{\partial w_i}$
- ▶ Better optimization algorithms to improve the training speed

Regular Gradient Descent Optimization (2/2)



```
n_output = 3
n_hidden = 4
n_features = 2
```

```
model = keras.models.Sequential()
model.add(keras.layers.Dense(n_hidden, input_shape=(n_features,), activation="relu"))
model.add(keras.layers.Dense(n_output, activation="softmax"))
```

```
model.compile(loss="sparse_categorical_crossentropy", optimizer="sgd", metrics=["accuracy"])
model.fit(X_train, y_train, epochs=30)
```

Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam Optimization

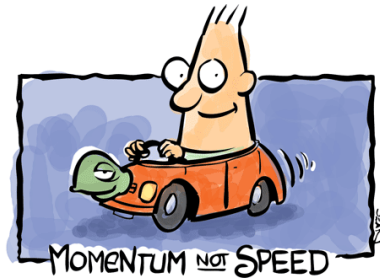


- ▶ **Momentum**
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization



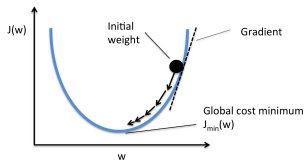
Momentum (1/3)

- ▶ **Momentum** is a concept from physics: an **object in motion** will have a **tendency to keep moving**.
- ▶ It measures the **resistance to change in motion**.
 - The **higher momentum** an object has, the harder it is to stop it.



Momentum (2/3)

- ▶ This is the very simple idea behind **momentum optimization**.
- ▶ We can see the **change in the parameters w** as **motion**: $w_i^{(next)} = w_i - \eta \frac{\partial J(w)}{\partial w_i}$
- ▶ We can thus use the concept of momentum to give the **update process** a **tendency to keep moving** in the same direction.
- ▶ It can help to **escape from bad local minima pits**.





Momentum (3/3)

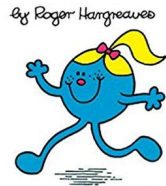
- ▶ Regular gradient descent optimization: $w_i^{(\text{next})} = w_i - \eta \frac{\partial J(w)}{\partial w_i}$
- ▶ **Momentum optimization** cares about what **previous gradients** were.
- ▶ At each iteration, it adds the **local gradient** to the **momentum vector** m .

$$m_i = \beta m_i + \eta \frac{\partial J(w)}{\partial w_i}$$
$$w_i^{(\text{next})} = w_i - m_i$$

- ▶ β is called **momentum**, and it is between 0 and 1.

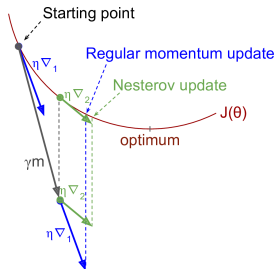
```
optimizer = keras.optimizers.SGD(lr=0.001, momentum=0.9)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization



Nesterov Momentum (1/2)

- ▶ **Nesterov Momentum** is a **small variant** to **Momentum optimization**.
- ▶ **Faster** than vanilla **Momentum optimization**.
- ▶ ∇_1 represents the **gradient of the cost function** measured at the **starting point w** , and ∇_2 represents the gradient at the point located at $w + \beta m$.





Nesterov Momentum (2/2)

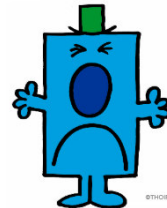
- ▶ Measure the gradient of the cost function slightly ahead in the direction of the momentum (not at the local position).

$$\mathbf{m}_i = \beta \mathbf{m}_i + \eta \frac{\partial J(\mathbf{w} + \beta \mathbf{m})}{\partial \mathbf{w}_i}$$
$$\mathbf{w}_i^{(\text{next})} = \mathbf{w}_i - \mathbf{m}_i$$

```
optimizer = keras.optimizers.SGD(lr=0.001, momentum=0.9, nesterov=True)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```

Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ **AdaGrad**
- ▶ RMSProp
- ▶ Adam optimization





AdaGrad (1/2)

- ▶ **AdaGrad** keeps track of a **learning rate** for **each parameter**.
- ▶ Adapts the **learning rate** over time (**adaptive learning rate**).
- ▶ Decays the learning rate **faster** for **steep dimensions** than for dimensions with **gentler slopes**.



AdaGrad (2/2)

- ▶ For each feature w_i , we do the following steps:

$$s_i = s_i + \left(\frac{\partial J(w)}{\partial w_i}\right)^2$$
$$w_i^{(\text{next})} = w_i - \frac{\eta}{\sqrt{s_i + \epsilon}} \frac{\partial J(w)}{\partial w_i}$$

```
optimizer = keras.optimizers.Adagrad(lr=0.001)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ **RMSPProp**
- ▶ Adam optimization





RMSProp (1/2)

- ▶ AdaGrad often stops too early when training neural networks.
- ▶ The learning rate gets scaled down so much that the algorithm ends up stopping entirely before reaching the global optimum.
- ▶ The RMSProp fixed the AdaGrad problem.
- ▶ It is like the AdaGrad problem, but accumulates only the gradients from the most recent iterations (not from the beginning of training).

RMSProp (2/2)

- ▶ For each feature w_i , we do the following steps:

$$s_i = \beta s_i + (1 - \beta) \left(\frac{\partial J(w)}{\partial w_i} \right)^2$$
$$w_i^{(\text{next})} = w_i - \frac{\eta}{\sqrt{s_i + \epsilon}} \frac{\partial J(w)}{\partial w_i}$$

```
optimizer = keras.optimizers.RMSprop(lr=0.001, rho=0.9)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization

By Roger Hargreaves





Adam Optimization (1/3)

- ▶ Adam (Adaptive moment estimation) combines the ideas of Momentum optimization and RMSProp.
- ▶ Like Momentum optimization, it keeps track of an exponentially decaying average of past gradients.
- ▶ Like RMSProp, it keeps track of an exponentially decaying average of past squared gradients.

Adam Optimization (2/3)

1. $m^{(\text{next})} = \beta_1 m + (1 - \beta_1) \nabla_w J(w)$
2. $s^{(\text{next})} = \beta_2 s + (1 - \beta_2) \nabla_w J(w) \otimes \nabla_w J(w)$
3. $m^{(\text{next})} = \frac{m}{1 - \beta_1^T}$
4. $s^{(\text{next})} = \frac{s}{1 - \beta_2^T}$
5. $w^{(\text{next})} = w - \eta m \oslash \sqrt{s + \epsilon}$

- ▶ \otimes and \oslash represent the **element-wise multiplication and division**.
- ▶ **Steps 1, 2, and 5**: similar to both **Momentum optimization** and **RMSPProp**.
- ▶ **Steps 3 and 4**: since **m** and **s** are initialized at 0, they will be biased toward 0 at the beginning of training, so these two steps will help **boost m** and **s** at the beginning of training.



Adam Optimization (3/3)

```
optimizer = keras.optimizers.Adam(lr=0.001, beta_1=0.9, beta_2=0.999)
model.compile(loss="sparse_categorical_crossentropy", optimizer=optimizer, metrics=["accuracy"])
```

Summary

Summary

- ▶ Overfitting
 - Early stopping, l_1 and l_2 regularization, max-norm regularization
 - Dropout, data augmentation
- ▶ Vanishing gradient
 - Parameter initialization, nonsaturating activation functions
 - Batch normalization, gradient clipping
- ▶ Training speed
 - Momentum, nesterov momentum, AdaGrad
 - RMSProp, Adam optimization





Reference

- ▶ Ian Goodfellow et al., Deep Learning (Ch. 7, 8)
- ▶ Aurélien Géron, Hands-On Machine Learning (Ch. 11)

Questions?